

# Leptons, quarks, and their antiparticles from a phase-space perspective

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**Abstract.** It is argued that antiparticles may be interpreted in macroscopic terms without explicitly using the concept of time and its reversal. The appropriate framework is that of nonrelativistic phase space. It is recalled that a quantum version of this approach leads also, alongside the appearance of antiparticles, to the emergence of ‘internal’ quantum numbers identifiable with weak isospin, weak hypercharge and colour, and to the derivation of the Gell-Mann-Nishijima relation, while simultaneously offering a preonless interpretation of the Harari-Shupe rishon model. Furthermore, it is shown that - under the assumption of the additivity of canonical momenta - the approach entails the emergence of string-like structures resembling mesons and baryons, thus providing a different starting point for the discussion of quark unobservability.

*‘It is utterly beyond our power to measure the changes of things by time (...) time is an abstraction at which we arrive by means of the changes of things; made because we are not restricted to any one definite measure, all being interconnected.’*

Ernst Mach - [1]

## 1. Introduction

### 1.1. Charge conjugation and time

In the Standard Model (SM), elementary particles are grouped into multiplets of various symmetry groups such as  $SU(2)$ ,  $SU(3)$ , etc. Particles and antiparticles belong then to complex conjugate representations (e.g. when coloured quarks are assigned to representation  $\mathbf{3}$  of  $SU(3)_C$ , the antiquarks are assigned to  $\mathbf{3}^*$ ). With the standard particle theory formulated on the background of classical space and time, the concepts of complex conjugation and time reversal are closely related. Accordingly, Stückelberg and Feynman interpret the antiparticles as particles ‘moving backwards in time’.

The spacetime-based description of Reality provided by the Standard Model is very successful. Yet, there are many questions that go beyond what SM was devised to answer. These include: why the physical quantities that enter into the SM as parameters have their specific values (masses of elementary particles or mixing angles between particle generations), what is the origin of various internal quantum numbers, or how to describe both elementary particles and gravity in a single framework.

The latter issue suggests that the standard particle theory should give way to an approach in which spacetime serves no longer as a background but becomes a dynamical structure, in line with general relativity ideas. In fact, many physicists have argued that we need an approach in which macroscopic classical time and space emerge - in the limit of a large interconnected structure - out of a simple timeless and alocal quantum level [2],[3]. The description of this level is still expected to be complex, as quantum descriptions are supposed to be [4].

Can one interpret then classically the expected presence of particles and antiparticles without explicitly using the concept of macroscopic time? In order to deal with this question, we find it appropriate to recall first how the classical concept of background time was originally introduced.

### 1.2. Time and change

In classical approaches with background time, position is viewed as a function of time  $\mathbf{x}(t)$ , while its change  $\delta\mathbf{x}$  is thought of as occurring in time, ‘*due*’ to its increase:  $\delta\mathbf{x} = \frac{d\mathbf{x}}{dt}\delta t$ . However, a deeper insight undermines this concept of background time. Rather, it views time as an effective parameter that somehow parametrizes *change*, as stated by Ernst Mach (see motto).

In [5] Barbour described how energy conservation (in an isolated system) serves as a key ingredient that leads to a definition of an increment of ephemeris time through observed changes:

$$\delta t = \sqrt{\frac{\sum_i m_i (\delta \mathbf{x}_i)^2}{2(E - V)}}, \quad (1)$$

where  $\delta\mathbf{x}_i$  are measured changes in the positions of astronomical bodies,  $E$  is total (and fixed) energy and  $V$  is the gravitational potential of all interacting bodies of the system. Thus, the (astronomical) time is defined by change, not vice versa. A more neutral expression of the relation between an increment of time  $\delta t$  and changes of position  $\delta\mathbf{x}_i$  is that the two are correlated. It is then up to us to decide which of the two alternative formulations to choose: change in (background) time, or time defined from (observed) change.

Now, in full analogy with Eq. (1), it is natural to consider

$$\delta t \mathbf{P} = \sum_i m_i \delta \mathbf{x}_i, \quad (2)$$

where  $\mathbf{P}$  is the total (and conserved) momentum of the system. For the sake of our discussion, we restrict the above equation to the one-particle case

$$\mathbf{p} \delta t = m \delta \mathbf{x}, \quad (3)$$

and observe that, again, one may view this relation in two ways:

- 1) either with momentum  $\mathbf{p}$  calculated from the change of position  $\delta\mathbf{x}$  in a given increment of (background) time  $\delta t$ , or
- 2) with time increment  $\delta t$  calculated from given  $\mathbf{p}$  and  $\delta\mathbf{x}$ .

The latter standpoint - in which momentum is not calculated from a change in position but assumed as independent of position - is familiar from the Hamiltonian formalism, in which momenta and positions are treated as independent variables. Thus, the idea of time induced by change should be expressible in the language of phase space in which the macroscopic arena is not 3- but 6- dimensional. Furthermore, with independent  $\mathbf{p}$  and  $\mathbf{x}$  one may consider various independent transformations of  $\mathbf{p}$  and  $\mathbf{x}$ , for example, study the symmetries of the time-defining equation Eq.(3). Then, standard 3D reflection corresponds to  $(\mathbf{p}, \mathbf{x}) \rightarrow (-\mathbf{p}, -\mathbf{x})$  (and leaves

time untouched), while the operation  $(\mathbf{p}, \mathbf{x}) \rightarrow (\mathbf{p}, -\mathbf{x})$  leads to time reflection:  $t \rightarrow -t$ .

Moving now to the quantum description we observe that the canonical commutation relations naturally involve the imaginary unit (we use units in which  $\hbar = 1$ ):

$$[x_k, p_l] = i\delta_{kl}. \quad (4)$$

In spite of containing  $i$ , Eq. (4) does not involve time explicitly. Consequently, since charge and complex conjugations are related, it should be possible to give an interpretation to the particle-antiparticle degree of freedom using the phase-space concepts of positions and momenta alone, i.e. without referring to the concept of explicit time. Charge conjugation should then be seen not only as connected to time reversal, but also as just one of several possible transformations in phase space.

## 2. Noncommuting phase space

In the standard picture with background spacetime there is a connection between the properties of the background (e.g. under 3D rotation) and the existence of the ‘spatial’ quantum numbers (e.g. spin). Therefore, if one is willing to enlarge the arena to that of phase space, with independent position and momentum coordinates, one might expect the appearance of additional quantum numbers [6]. From the point of view of the standard (3D space + time) formalism, such quantum numbers would necessarily appear ‘internal’.

### 2.1. Born reciprocity

The issue of a possible relation between particle properties (such as quantum numbers or masses), and the concept of phase space was of high concern already to Max Born. In his 1949 paper [7] he discussed the difference between the concepts of position and momentum for elementary particles and noted that the notion of mass appears in the relation  $p^2 = m^2$ , while  $x^2$ , the corresponding invariant in coordinate space (with  $x^2$  of atomic dimensions), does not seem to enter in a similar relation. At the same time, Born stressed that various laws of nature such as

$$\begin{aligned} \dot{x}_k &= \frac{\partial H}{\partial p_k}, & \dot{p}_k &= -\frac{\partial H}{\partial x_k}, \\ [x_k, p_l] &= i\delta_{kl}, \\ L_{kl} &= x_k p_l - x_l p_k, \end{aligned} \quad (5)$$

are invariant under the ‘reciprocity’ transformation:

$$x_k \rightarrow p_k, \quad p_k \rightarrow -x_k. \quad (6)$$

Noting that relation  $p^2 = m^2$  is not invariant under this transformation, he concluded: ‘*This lack of symmetry seems to me very strange and rather improbable*’.

The simplest phase-space generalization of the 3D (nonrelativistic) concepts of rotation and reflection is obtained with a symmetric treatment of the two  $O(3)$  invariants,  $\mathbf{x}^2$  and  $\mathbf{p}^2$ , via their addition (obviously, this procedure requires an introduction of a new fundamental constant of Nature of dimension [momentum/distance]):

$$\mathbf{x}^2 + \mathbf{p}^2. \quad (7)$$

Eq.(7) is invariant under  $O(6)$  transformations and Born reciprocity in particular.

We may now treat  $\mathbf{x}$  and  $\mathbf{p}$  as operators, and require their commutators to be form invariant. The original  $O(6)$  symmetry is then reduced to  $U(1) \otimes SU(3)$ . The appearance of the symmetry group present in the Standard Model leads us to ask the question whether phase-space symmetries could possibly lie at the roots of SM symmetries. As shown in [6] and also argued below, this seems quite possible.

**Table 1.** Decomposition of eigenvalues of  $Y$  into eigenvalues of its components.

colour	0	1	2	3
$Y$	$-1$	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$
$Y_1$	$-\frac{1}{3}$	$-\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$
$Y_2$	$-\frac{1}{3}$	$+\frac{1}{3}$	$-\frac{1}{3}$	$+\frac{1}{3}$
$Y_3$	$-\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	$-\frac{1}{3}$

## 2.2. Dirac linearization

Our present theories are standardly divided into classical and quantum ones. Yet, as stressed by Finkelstein, the latter should be more appropriately regarded as belonging to a mixed, classical-quantum type. In Finkelstein's view, the classical (c), and classical-quantum (cq) theories should give way to a purely quantum (q) approach in which infinities would not appear and the concept of infinite and infinitely divisible background space would no longer exist [3]. This view is clearly in line with the arguments for a background-independent approach to quantum gravity.

The Dirac linearization prescription may be thought of as a procedure that has led us from the classical description of Nature to part of its quantum description. Indeed, the linearization of  $\mathbf{p}^2$  leads to the appearance of Pauli matrices, which describe spin at the quantum level. It is therefore of great interest to apply Dirac's idea to the phase-space invariant of Eq.(7). Using anticommuting matrices  $A_k$  and  $B_k$  ( $k = 1, 2, 3$ ) (for more details, see [6]) one finds

$$(\mathbf{A} \cdot \mathbf{p} + \mathbf{B} \cdot \mathbf{x})(\mathbf{A} \cdot \mathbf{p} + \mathbf{B} \cdot \mathbf{x}) = (\mathbf{p}^2 + \mathbf{x}^2) + \sum_1^3 \sigma_k \otimes \sigma_k \otimes \sigma_3 \equiv R + R^\sigma. \quad (8)$$

The first term on the r.h.s.,  $R = \mathbf{p}^2 + \mathbf{x}^2$ , appears thanks to the anticommutation properties of  $A_k$  and  $B_l$ . The other term,  $R^\sigma$ , appears because  $x_k$  and  $p_k$  do not commute. These two terms sum up to a total  $R^{tot} = R + R^\sigma$ .

When viewed from Finkelstein's perspective, the invariant

$$\mathbf{A} \cdot \mathbf{p} + \mathbf{B} \cdot \mathbf{x} \quad (9)$$

connects then the cq-level of phase space (noncommuting positions  $\mathbf{x}$  and momenta  $\mathbf{p}$ ) with the presumably purely q-level structure: the Clifford algebra built from matrices  $\mathbf{A}$  and  $\mathbf{B}$ .

## 3. Quantization

### 3.1. Gell-Mann-Nishijima relation

Just as  $R$  is quantized, so is  $R^\sigma$ . Thus, we have to find the eigenvalues of  $R^\sigma$ . For better correspondence with the standard definitions of internal quantum numbers, we introduce operator  $Y$ :

$$Y \equiv \frac{1}{3} R^\sigma B_7 = \frac{1}{3} \sum_1^3 \sigma_k \otimes \sigma_k \otimes \sigma_0 \equiv \sum_1^3 Y_k. \quad (10)$$

where  $B_7 = iA_1A_2A_3B_1B_2B_3$  is the 7-th anticommuting element of the Clifford algebra.

Since  $Y_k$  commute among themselves, they may be simultaneously diagonalized. The eigenvalues of  $Y_k$  ( $k = 1, 2, 3$ ) are  $\pm 1/3$ . The resulting pattern of possible eigenvalues of  $Y$  is shown in Table 1.

**Table 2.** Rishon structure of leptons and quarks with  $I_3 = +1/2$ .

	$\nu_e$	$u_R$	$u_G$	$u_B$	$e^+$	$\bar{d}_R$	$\bar{d}_G$	$\bar{d}_B$
	$VVV$	$VTT$	$TVT$	$TTV$	$TTT$	$TVV$	$VTV$	$VVT$
$Q$	0	$+\frac{2}{3}$	$+\frac{2}{3}$	$+\frac{2}{3}$	+1	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$
$Y$	-1	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	+1	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$

In [6] a conjecture was put forward that the electric charge  $Q$  is proportional to operator  $R^{tot}B_7$ , evaluated for the lowest level of  $R$ , i.e.:

$$Q = \frac{1}{6}(R_{lowest} + R^\sigma)B_7 = I_3 + \frac{Y}{2}, \quad (11)$$

where  $R_{lowest} = (\mathbf{p}^2 + \mathbf{x}^2)_{lowest} = 3$ , and  $I_3 = B_7/2$ .

The above equation is known under the name of the Gell-Mann-Nishijima relation (with  $I_3$  of eigenvalues  $\pm 1/2$  known as weak isospin and  $Y$  of eigenvalues  $-1, +1/3$  known as weak hypercharge) and is considered to be a law of nature. It summarizes the pattern of charges of all eight leptons and quarks from a single SM generation. In the phase-space approach it is *derived* as a consequence of phase-space symmetries.

### 3.2. Harari-Shupe rishons

As shown in [6], the pattern in which the weak hypercharge  $Y$  is built out of ‘partial hypercharges’  $Y_k$  corresponds *exactly* to the pattern in which electric charges are built in the Harari-Shupe (HS) model of quarks and leptons [8]. The HS approach describes the structure of a SM generation with the help of a composite model: it builds all eight fermions of a single generation from two spin-1/2 ‘rishons’  $V$  and  $T$  of charges 0 and  $+1/3$ . The proposed structure of leptons and quarks is shown in Table 2.

Our phase-space approach not only reproduces exactly the successful part of the rishon structure, but it also removes all the main shortcomings of the HS model. In particular, the approach is *preonless*, i.e. the phase-space ‘rishons’ are components of charge (hypercharge) only, with *no interpretation in terms of spin 1/2 subparticles*. Thus, there is no problem of rishon confinement. Consequently, our leptons and quarks are viewed as pointlike, in perfect agreement with the experimental knowledge. In summary, the phase-space approach explains the origin of the observed symmetries without introducing any subparticles.

One might object that the nontrivial combination of spatial and internal symmetries is forbidden by the Coleman-Mandula no-go theorem [9]. Yet, this theorem is neatly evaded by our construction: the theorem works at the S-matrix level, while quarks are to be confined, as we expect and as will be argued later on. Furthermore, no additional dimensions (standardly understood) have actually been added in our framework. The only change was a shift in the conceptual point of view: instead of a picture based on 3D space and time, we decided to view the world in terms of a picture based on the 6D arena of canonically conjugated positions and momenta.

## 4. Compositeness and additivity

The linearized phase-space approach suggests that the Clifford algebra of nonrelativistic phase space occupies an important place in our description of leptons and quarks (see also [10]). Thus, one would like to use it also in a description of composite systems, in particular in the description

of hadrons as composite systems of quarks. Unfortunately, it is not clear how to achieve this goal. Yet, the q-level construction in question must be related to the c-level description of Nature, and, consequently, should be interpretable at the purely classical level. Indeed, as Niels Bohr said: ‘*However far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms*’. In fact, Eq.(9) provides the required connection through which transformations between leptons, quarks, and their antiparticles are related to those in phase space.

Now, in any description of composite systems, be it at the classical or the quantum level, an important ingredient is provided by the tacitly assumed concept of additivity. Indeed, additivity is assumed both at the quark level (e.g. additivity of spins or flavour quantum numbers), and at the classical level (e.g. additivity of momenta). The question then appears: can this property of the additivity of momenta at the classical level be somehow used to infer about the properties of such q-level objects as our quarks when these are viewed from the macroscopic classical perspective.

The relevant basic macroscopic observation is that for any direction in 3D space, and completely regardless of what happens in the remaining two directions, we have the *additivity of physical momenta of any number of ordinary particles or antiparticles, quite irrespectively of their internal quantum numbers* (e.g.  $P_z = \sum_i p_{iz}$ ). (By ordinary particles we mean those that can be observed *individually*, such as leptons and hadrons, but not quarks.) Note that the positions of ordinary particles are not additive in such a simple way, since a composite object is best described in terms of its center-of-mass coordinates:  $X_z = \sum_i m_i x_{iz} / \sum_i m_i$ . In the following, we shall present the connections between lepton-quark and phase-space transformations and discuss their implications for the concept of the additivity of momenta.

#### 4.1. Charge conjugation

In a quantum description the transition from particles to antiparticles is effected by complex conjugation. Consider now a system of particles in which some are being transformed into antiparticles. We want to find the related transformation in phase space. We note that in order to preserve the principle of the additivity of physical momenta, one is not allowed to change the momenta of any of the particles being transformed. The invariance of  $[x_k, p_l] = i\delta_{kl}$  requires then that

$$p_k \rightarrow p_k, \quad x_k \rightarrow -x_k, \quad i \rightarrow -i. \quad (12)$$

Using the invariance of Eq.(9), the ensuing transformations of  $A_k$  and  $B_k$ , and the definition of  $Q$ ,  $I_3$ , and  $Y$ , one can check [11] that in this way we are indeed led from particles to antiparticles. Thus, in the phase-space picture, with time regarded as secondary, the antiparticles are related to particles via  $i \rightarrow -i$  combined with the reflection of position space (with the time reflection  $t \rightarrow -t$  induced via the invariance of Eq.(3)).

#### 4.2. Isospin reversal

Similarly, one checks [11] that isospin reversal  $I_3 \rightarrow -I_3$  corresponds to the transformation  $A_k \rightarrow A_k, B_k \rightarrow -B_k$ , and  $i \rightarrow i$ , and, consequently, to  $p_k \rightarrow p_k$  and  $x_k \rightarrow -x_k$ . Yet, this is not the same as charge conjugation since in this case, with  $i \rightarrow +i$ , the momentum-position commutation relations do not stay invariant.

In summary, the phase-space representations of both the particle-antiparticle transformation and of the isospin reversal may be (and have been) chosen in a way that does not affect the momenta of any of the particles, thus preserving their additivity.

#### 4.3. Lepton-to-quark transformations

Transformations from the lepton to the quark sector (in particular, the change  $Y = -1 \rightarrow Y = +1/3$ ) require the use of  $SO(6)$  rotations going outside those generated by the familiar  $1 + 8$  generators of  $U(1) \times SU(3)$ . The remaining six of the fifteen  $SO(6)$  generators form two  $SU(3)$  triplets, of which only one actually leads to transformations of some  $A_k$  into  $B_m$ , while keeping  $i$  and  $I_3$  fixed [6].

Under these transformations,  $\mathbf{A}$  is changed into some  $\mathbf{A}^{Qn}$ , and  $\mathbf{B}$  into some  $\mathbf{B}^{Qn}$ , with  $n = 1, 2, 3$  being the colour index [11]. For the transformed elements  $\mathbf{A}^{Qn}$  (and similarly for  $\mathbf{B}^{Qn}$ ) one can then choose the following (not unique) representation :

$$A^Q \equiv \begin{bmatrix} \mathbf{A}^{Q1} \\ \mathbf{A}^{Q2} \\ \mathbf{A}^{Q3} \end{bmatrix} = \begin{bmatrix} A_1 & -B_3 & +B_2 \\ +B_3 & A_2 & -B_1 \\ -B_2 & +B_1 & A_3 \end{bmatrix}, \quad (13)$$

with the quark *canonical momenta*  $\mathbf{P}^{Qn}$  (and likewise, for quark canonical positions  $\mathbf{X}^{Qn}$ ), obtained from the condition of the invariance of Eq.(9):

$$P^Q \equiv \begin{bmatrix} \mathbf{P}^{Q1} \\ \mathbf{P}^{Q2} \\ \mathbf{P}^{Q3} \end{bmatrix} = \begin{bmatrix} p_1^1 & -x_3^1 & +x_2^1 \\ +x_3^2 & p_2^2 & -x_1^2 \\ -x_2^3 & +x_1^3 & p_3^3 \end{bmatrix}. \quad (14)$$

The above forms of  $A^Q$  and  $P^Q$  remain unchanged for quarks of opposite isospin.

For the antiquarks, the relevant forms are

$$A^{\bar{Q}} = \begin{bmatrix} A_1 & +B_3 & -B_2 \\ -B_3 & A_2 & +B_1 \\ +B_2 & -B_1 & A_3 \end{bmatrix}, \quad (15)$$

and

$$P^{\bar{Q}} = \begin{bmatrix} p_1^1 & +x_3^1 & -x_2^1 \\ -x_3^2 & p_2^2 & +x_1^2 \\ +x_2^3 & -x_1^3 & p_3^3 \end{bmatrix}. \quad (16)$$

Again, these forms are independent of isospin.

As expected, the difference between quarks and antiquarks is represented by a change in sign in front of physical positions entering into the definitions of quark (antiquark) canonical momenta. *If* additivity of canonical momenta of quarks is a proper generalization of the additivity of physical momenta for leptons and other individually observable particles, then - on account of the relative (positive and negative) signs between position components in (14) and (16) - this additivity (separately in each of the relevant phase-space directions) leads to translationally invariant string-like expressions for quark-antiquark and three-quark systems (i.e.  $x_2^1(q) - x_2^1(\bar{q})$  for  $q\bar{q}$  and  $(x_1^3(q_3) - x_1^2(q_2), x_2^1(q_1) - x_2^3(q_3), x_3^2(q_2) - x_3^1(q_1))$  for  $q_1q_2q_3$ ), but not for  $qq$  or  $qqqq$  systems. Additivity of canonical momenta leads therefore to the formation of ‘mesons’ and ‘baryons’ only.

In principle, the chain of arguments leading to Eq. (13,14) could involve ordinary reflections, e.g.  $(A_1, -B_3, +B_2) \rightarrow (A_1, +B_3, +B_2)$ , *before* putting the latter expression and its cyclic counterparts together into a matrix form similar to (13) (‘grouping’). The corresponding phase-space counterparts would then look somewhat different, i.e.  $(p_1, -x_3, +x_2) \rightarrow (p_1, +x_3, +x_2)$  etc. The translational invariance of three-quark systems could then not be achieved by simply adding the appropriate canonical momenta of different quarks, because all physical position

coordinates would enter an analog of (14) with positive signs. The point, however, is that one *may* choose the ordering of the operations of grouping and reflection in such a way that - by the simple procedure of addition - the translational invariance can be achieved at all. Furthermore, it seems nontrivial that this requires collaboration of phase-space representatives of quarks of three different colours. After all, the latter were originally defined - *in a way seemingly independent of the concept of additivity* - via a diagonalization procedure performed at the Clifford algebra level.

In other words, the q-level structure of coloured quark charges corresponds to a specific picture in the macroscopic arena. According to this picture, individual quarks are not observable at the classical level since their individual canonical momenta are not translationally invariant. On the other hand, translational invariance may be restored via the collaboration of quarks of different colours. When viewed from our classical point of view, the resulting composite systems possess standard particle-like properties, while at the same time exhibiting internal string-like features. Thus, quark unobservability is supposed to be connected to the very emergence and nature of space and time. Speaking more precisely, quarks are supposed to be unobservable because space and phase space are most probably just convenient classical abstractions, into the descriptive corset of which we try to force various pieces of Reality.

The often-used argument that ‘space is standard’ at the distances a few orders of magnitude smaller than proton’s size is not sufficiently sound. After all, the existence of long-distance nonlocal quantum correlations indicates that - at least for some purposes - our classical spacetime concepts (into which we try to force our descriptions of elementary particles) are inadequate at much larger scales. In fact, statements about an ‘unchanged nature of space’ at a distance of  $10^{-18}$  m or so follow from the success of the Standard Model (a cq-level theory) and are strictly valid only *within* the description it provides, not outside of it (e.g. not in a q-level theory in which space is to be an emergent concept only).

The above discussion suggests that the phase-space approach has the capability of describing the phenomenon of quark unobservability in a way seemingly different from the SM flux-tube picture of confinement. In fact, however, the phase-space approach does not have to be in conflict with the latter picture, just as the Faraday picture of ‘real’ fundamental field lines is not in conflict with the Maxwell concept of fields. Rather, we regard the idea of the linearized phase space as offering a possible q-level starting point. Our discussion is based on the invariance of Eq. (9) which provides a link between q- and cq- (c- ) levels of description. Obviously, however, Eq. (9) does not specify how the macroscopic background phase space actually emerges from the underlying quantum level. Hence, at present there is no way to compare our ideas directly with the standard, background-dependent, QCD-based picture of confinement.

## 5. Conclusions

The linearized phase-space approach differs markedly from the standard frameworks, in particular from the  $SU(5)$ -based unifications. A brief comparison of some differences between the two schemes is given in Table 3. In the author’s opinion, the  $SU(5)$  approach lacks a solid philosophical background. On the other hand, the phase-space approach, although overly simplistic, satisfies an important philosophical condition: the necessity to connect the q-level description of elementary particles to the c-level description of the macroscopic world. One can hear the echoes of this condition in the words of Roger Penrose, who stated in [12]: ‘*I do not believe that a real understanding of the nature of elementary particles can ever be achieved without a simultaneous deeper understanding of the nature of spacetime.*’

The phase-space approach provides a possible theoretical explanation of the structure of a single generation of the Standard Model. It gives us a tentative (pre)geometric interpretation of the origin of the Gell-Mann-Nishijima relation. It reproduces the structure of the Harari-Shupe preon model without actually introducing any preons at all, in line with the standard pointlike



**Table 3.** Comparison with  $SU(5)$ .

Simple group – $SU(5)$	Linearized phase space
$5^* + 10 (+1)$ representations	two replicas of 4 and $4^*$ of $SU(4)$
fair prediction for Weinberg angle	–
anomaly free	–
no connection to macroscopic arena	connection to macroscopic phase space
proton decay	protons are forever
–	Harari – Shupe reproduced

description of fundamental fermions. It touches on the question of the origin of time. It suggests that the phenomenon of quark unobservability is related to the very nature of space and time, which are regarded as emergent (or abstract) classical concepts only.

The picture offered by the Clifford algebra of nonrelativistic phase space need not be regarded as ‘the’ q-level approach. Rather, it should be thought of as a possible deeper layer of description only. However, it has encouraging features, which - as I believe - will show up at the classical level if derived ‘in an emergent way’ from any suitable q-level description. Let me therefore end by paraphrasing Penrose’s opinion: I do not believe that deeper understanding of elementary particles can be achieved without further studies of the proposed link between the elementary particles themselves and the properties and symmetries of nonrelativistic phase space.

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## References

- [1] Mach E 1960 (English translation) *The Science of Mechanics* (Chicago: Open Court)
- [2] Penrose R 1971 Angular Momentum: an Approach to Combinatorial Space-Time *Quantum Theory and Beyond* ed T Bastin (Cambridge: Cambridge Univ. Press) p 151
- Wheeler J A 1980 Pregelometry: Motivations and Prospects *Proceedings of a Symposium on Quantum Theory and Gravitation* ed A R Marlow (New York: Academic Press) p 1
- Rovelli C 1991 *Phys. Rev. D* **43** 442
- [3] Finkelstein D 1972 *Phys. Rev. D* **5** 320
- [4] Barbour J 1993 *Phys. Rev. D* **47** 5422
- [5] Barbour J 2003 *Lect. Notes Phys.* **633** 15
- Barbour J 2009 The nature of time. First prize of the FQXi *The Nature of Time* contest (arXiv:0903.3489)
- [6] Żenczykowski P 2007 *Acta Phys. Pol. B* **8** 2053, 2631
- Żenczykowski P 2008 *Phys. Lett. B* **660** 567
- Żenczykowski P 2009 *J. Phys. A* **42** 045204
- Żenczykowski P 2009 *J. Phys. Conf. Ser.* **174** 012032
- [7] Born M 1949 *Rev. Mod. Phys.* **21**, 463
- [8] Harari H 1979 *Phys. Lett. B* **6** 83
- Shupe M A 1979 *Phys. Lett. B* **86** 87
- [9] Coleman S, Mandula J 1967 *Phys. Rev.* **159** 1251
- [10] Hestenes D, Sobczyk G 1984 *Clifford Algebra to Geometric Calculus* (Dordrecht: D Reidel)
- Pavšič M 2003 *Found. Phys.* **33** 1277
- [11] Żenczykowski P 2010 *J. Theor. Phys.* **49** 2246
- [12] Penrose R 1968 Structure of Spacetime *Batelle Rencontres: 1967 Lectures in Mathematics and Physics* ed C M DeWitt and J A Wheeler (New York: Benjamin) p 121